<u>12.2 Vector Intro</u>

Def'n & Notation: A **vector** represents magnitude and direction.

We depict a vector with an arrow:

a. *magnitude* = "arrow length".
b. *initial point* = "tail of the arrow"
c. *terminal point* = "head of arrow"

We denote a vector according changes in x, y and z change from tail-to-head.

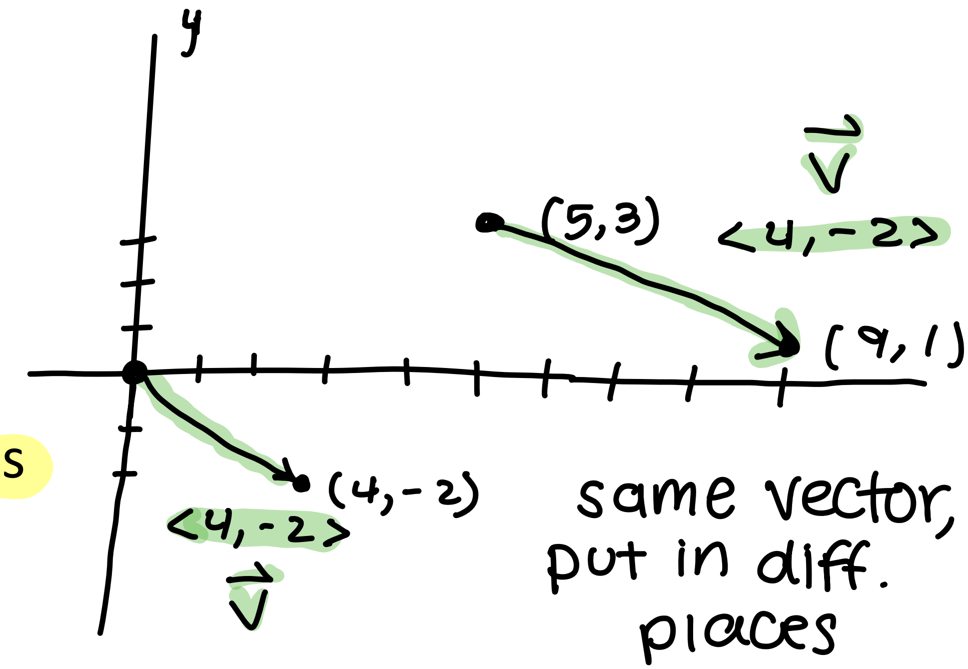
$$\boldsymbol{v} = \langle v_1, v_2, v_3 \rangle$$
$$= \langle \Delta x, \Delta y, \Delta z \rangle$$

In other words, if you move the vector so the tail is at the origin, then the head will be at the point (v_1, v_2, v_3)

Entry Task: Consider the 2D vector

$$\boldsymbol{v} = \langle 4, -2 \rangle \leftarrow \text{vector}$$

(a) Draw it with its tail at (5,3) ← Point
(b) Draw it with its tail at (0,0)



vectors designate changes

Quick Run of Basic Facts

(please also read the text!)

- Two vectors are equal if all components are equal.

- The **vector *from* A(a_1,a_2,a_3) to B(b_1,b_2,b_3)** is given by

$$\overrightarrow{AB} = < b_1 - a_1,\ b_2 - a_2,\ b_3 - a_3 >$$

head - tail

- We denote $\overbrace{\textbf{magnitude}}^{\text{length}}$ by

$$|\boldsymbol{v}| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}$$

$\underbrace{\text{distance}}$ formula

absolute value

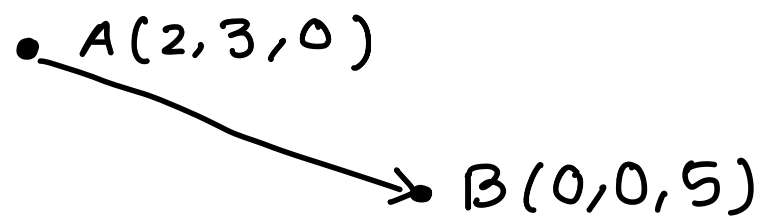
Examples

- If <a, 4> = <7, 4>, what is *a*? 7

- Find the vector depicted by drawing an arrow with tail at A(2,3,0) and head is at B(0,0,5)?

$$\overrightarrow{AB} = <0-2,\ 0-3,\ 5-0>$$

$$\boxed{\overrightarrow{AB} = <-2,\ -3,\ 5>}$$



- Give the magnitude of $\boldsymbol{v}$ = < 3, 2, 4>?

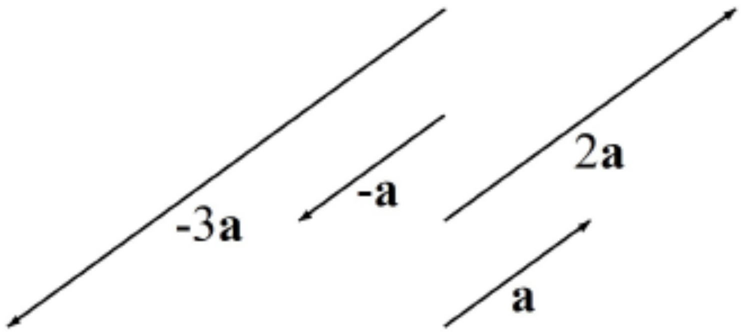
$$|V| = \sqrt{9+4+16} = \boxed{\sqrt{29}}$$

- **Scalar Multiplication**

If c is a constant, then we define

$$c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle,$$

which scales the magnitude by a factor of c.



- A **unit vector** has length one. Note: $\frac{1}{|v|}v$ = "unit vector in the same direction as **v**".

Visuals: https://www.math3d.org/W5yYV3uy

Examples

- Give the vector in the direction of $v = \langle 3, 2, 4 \rangle$ that is twice as long.

$2\vec{v} = 2\langle 3,2,4 \rangle$

$2\vec{v} = \langle 6,4,8 \rangle$

- Give the vector in the opposite direction of $v = \langle 3, 2, 4 \rangle$ that is three times as long.

negative

$-3\vec{v} = -3\langle 3,2,4 \rangle$

$-3\vec{v} = \langle -9,-6,-12 \rangle$

- Give the vector that is length one and in the direction of $v = \langle 3, 2, 4 \rangle$

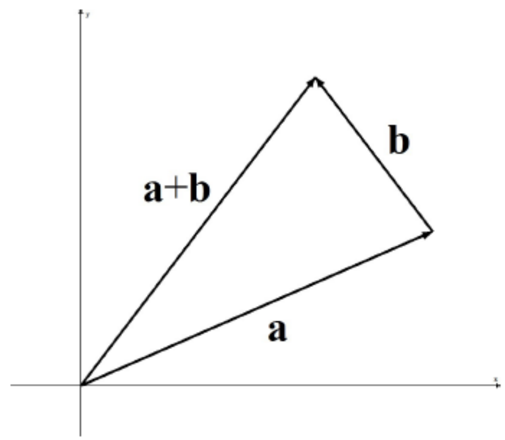
$|\vec{v}| = \sqrt{9+4+16} = \sqrt{29}$

$$\frac{1}{\sqrt{29}}\langle 3,2,4 \rangle = \boxed{\left\langle \frac{3}{\sqrt{29}}, \frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}} \right\rangle}$$

Vector Sum:

We define

$$\mathbf{v} + \mathbf{w} = \langle v_1, v_2, v_3\rangle + \langle w_1, w_2, w_3\rangle$$
$$= \langle v_1 + w_1, v_2 + w_2, v_3 + w_3\rangle$$



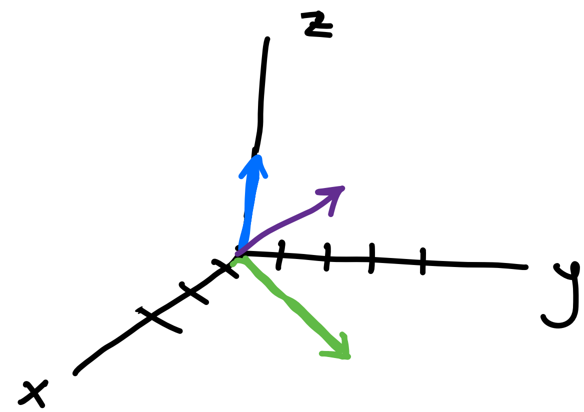
Example

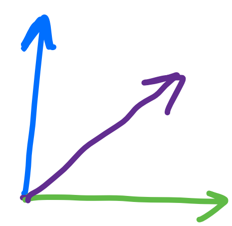
in/out right/left up/down

- Find and try to visualize the sum of $\boldsymbol{a} = \langle 3,4,0\rangle$ and $\boldsymbol{b} = \langle 0,0,2\rangle$.

$a+b = \langle 3+0, 4+0, 0+2\rangle$

$a+b = \langle 3,4,2\rangle$



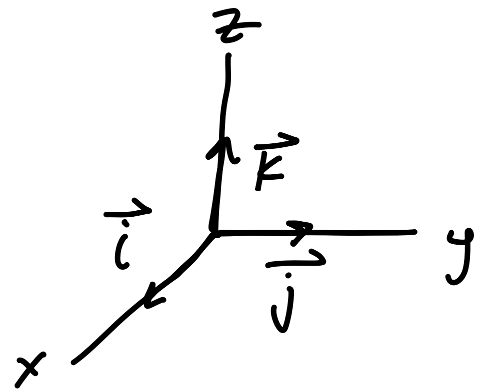


Side view

vector sum = resultant force

Visuals: https://www.math3d.org/W5yYV3uy

- Standard unit basis vectors:
  - **i** = < 1, 0, 0 > = $\vec{i}$
  - **j** = < 0, 1, 0 > = $\vec{j}$
  - **k** = < 0, 0, 1 > = $\vec{k}$



Examples:

- What vector is given by

  $$\boldsymbol{v} = 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}?$$

  $= 3<1,0,0> + 2<0,1,0> - <0,0,1>$

  $= <3,0,0> + <0,2,0> - <0,0,1>$

  $= \boxed{<3,2,-1>}$ $\longrightarrow$ also $3\vec{i} + 2\vec{j} - k$

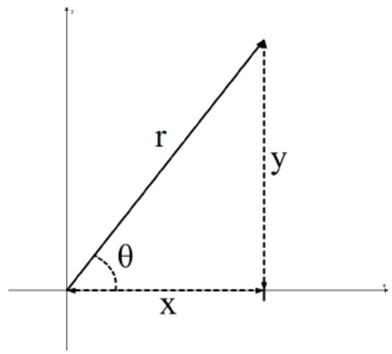
- What vector is given by

  $$\boldsymbol{v} = 2\boldsymbol{i} + \boldsymbol{k}?$$

  $= 2<1,0,0> + <0,0,1>$

  $= \boxed{<2,0,1>}$

- **Working in 2D (angles?):**
  If the angle, θ, and length, *r*, as shown



Remember,
$x = r\cos(\theta)$, $y = r\sin(\theta)$, $x^2 + y^2 = r^2$.

$$\vec{b} = \langle -300, 0 \rangle$$

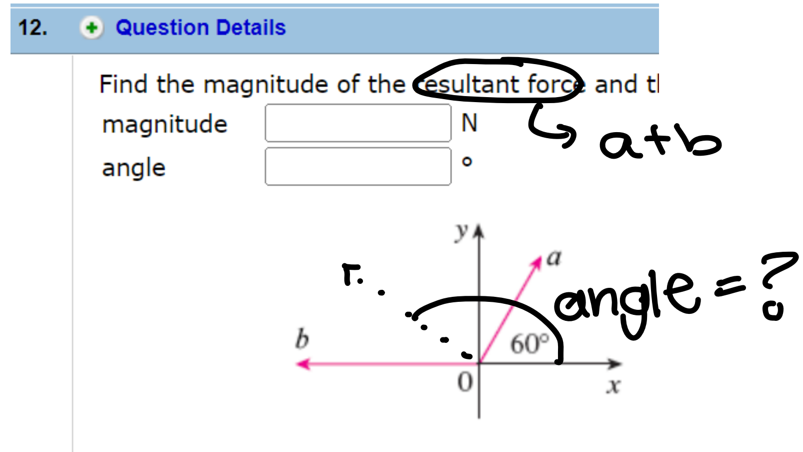
$$\vec{a} = \langle 100, 100\sqrt{3} \rangle$$

$$\vec{a} + \vec{b} = \langle -200, 100\sqrt{3} \rangle$$

magnitude $= \sqrt{200^2 + (100\sqrt{3})^2}$

angle = ?

Example (HW question)



12. Question Details

Find the magnitude of the resultant force and t

magnitude N

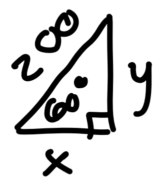
angle °

Given: |**a**| = 200 N, |**b**| = 300N.

Want:

Length of **a** + **b**?

Angle **a** + **b** makes with pos. x-axis?



$\cos(60°) = \frac{x}{200} = \frac{1}{2}(200) = 100$

$\sin(60°) = \frac{y}{200} = 200\left(\frac{\sqrt{3}}{2}\right) = 100\sqrt{3}$

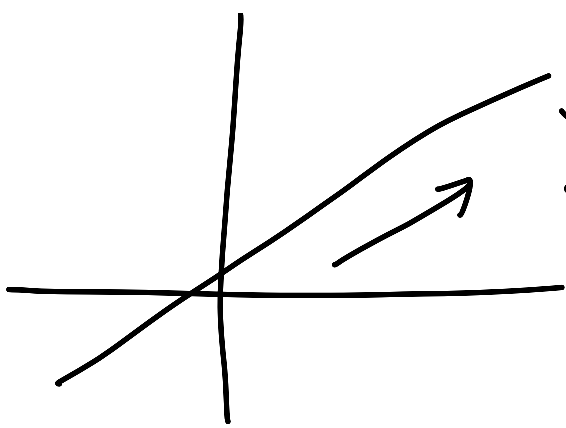
- **Working in 2D (slopes?):** If you want a vector that is **parallel to a line with slope *m***, then the vector < 1 , m > works!

Example (HW question):

13. Question Details

Find a unit vector that is parallel to the line tangent to the parabola $y = x^2$ at the point (3, 9).

unit vector parallel → divide by length!



$y=2x+1$

vector parallel to line

$<\Delta x, \Delta y>$

same slope! $\left(\frac{\Delta y}{\Delta x}\right)$

Visual: https://www.desmos.com/calculator/rfr5ajmxkt

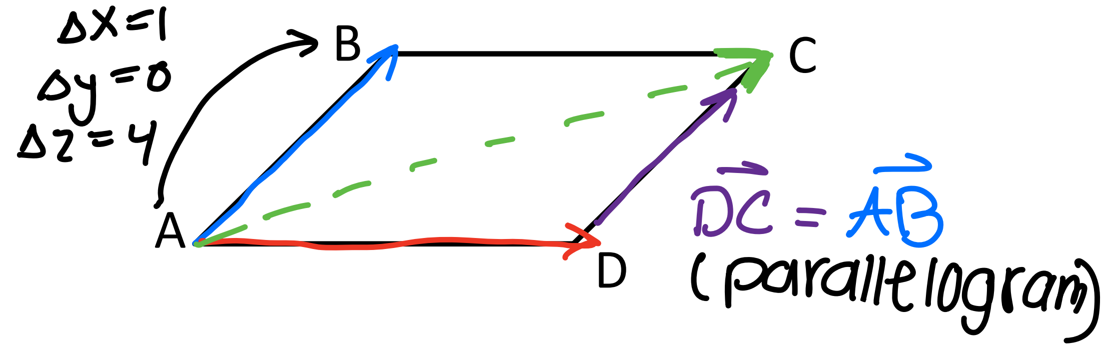
12.2 Old Exam Problem (*do this over and over again until it is easy!*)

The parallelogram shown has corners, A(1,1,2), B(2,1,6), C(?,?,?) and D(5,2,3)

(a) Find $\overrightarrow{AB} + \overrightarrow{AD}$.

(b) Find the coordinates of C.

(c) Find a unit vector that is parallel to $\overrightarrow{BD}$.



head-tail

$\overrightarrow{AB} = \langle 2-1, 1-1, 6-2 \rangle = \langle 1, 0, 4 \rangle$

$\overrightarrow{AD} = \langle 5-1, 2-1, 3-2 \rangle = \langle 4, 1, 1 \rangle$

$\overrightarrow{AB} + \overrightarrow{AD} = \boxed{\langle 5, 1, 5 \rangle} = \overrightarrow{AC}$

changes!

to get from point A to point C ...

$\boxed{C = (6, 2, 7)}$

length $= \sqrt{9+1+9} = \sqrt{19}$

$\overrightarrow{BD} = \langle 5-2, 2-1, 3-6 \rangle = \langle 3, 1, -3 \rangle$

$\boxed{\left\langle \frac{3}{\sqrt{19}}, \frac{1}{\sqrt{19}}, \frac{-3}{\sqrt{19}} \right\rangle}$

visual: https://www.math3d.org/43Ir397M

Preview - 12.3 Dot Products

We define

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Manipulation facts

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$$

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$$

$$\mathbf{a} \cdot \mathbf{a} = \mathrm{a}_1^2 + a_2^2 + a_3^2 = |\boldsymbol{a}|^2$$

The most important fact:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta),$$

where θ is the smallest angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. $(0 \leq \theta \leq \pi)$ TAIL-TO-TAIL!!

Most important consequence:

If **a** and **b** are orthogonal, then

$$\mathbf{a} \cdot \mathbf{b} = 0.$$